Chapter 9

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Linear Momentum and Collisions

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Lecture 01

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Linear Momentum and it's Conservation

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Momentum Analysis Models

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- Force and acceleration are related by Newton's second law.
- When force and acceleration vary by time, the situation can be very complicated.
- The techniques developed in this chapter will enable you to understand and analyze these situations in a simple way.
- Will develop momentum versions of analysis models for isolated and non-isolated systems
- These models are especially useful for treating problems that involve collisions and for analyzing rocket propulsion.

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Thought Experiment

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- An archer stands on frictionless ice and fires an arrow. What is the archer's velocity after firing the arrow?
  - Motion models such as a particle under constant acceleration cannot be used.
    - No information about the acceleration of the arrow
  - Model of a particle under constant force cannot be used.
    - No information about forces involved
  - Energy models cannot be used.
    - No information about the work or the energy (energies) involved
- A new quantity is needed – linear momentum.

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Linear Momentum

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Consider newton's second law:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

With constant mass.

The net force is equal to the change in the product $(m\vec{v})$ per unit time.

This product is called the linear momentum (or the momentum).

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Linear Momentum

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The linear momentum of a particle or an object of mass m moving with a velocity $\vec{v}$ is defined to be the product of the mass and velocity:

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity.

Its direction is the same as the direction of the velocity.

The SI units of momentum are:

$$kg.m/s$$

Momentum can be expressed in component form:

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

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Conservation of Linear Momentum

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Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

- The momentum of the system is conserved, not necessarily the momentum of an individual particle.
  - Avoid applying conservation of momentum to a single particle.
- This also tells us that the total momentum of an isolated system equals its initial momentum.

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Conservation of Momentum

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Conservation of momentum can be expressed mathematically in various ways:

$$\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = Constant$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

In component form, the total momenta in each direction are independently conserved.

$$\vec{p}_{1ix} + \vec{p}_{2ix} = \vec{p}_{1fx} + \vec{p}_{2fx}$$

$$\vec{p}_{1iy} + \vec{p}_{2iy} = \vec{p}_{1fy} + \vec{p}_{2fy}$$

$$\vec{p}_{1iz} + \vec{p}_{2iz} = \vec{p}_{1fz} + \vec{p}_{2fz}$$

Conservation of momentum can be applied to systems with any number of particles.

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Forces and Conservation of Momentum

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In conservation of momentum, there is no statement concerning the types of forces acting on the particles of the system.

The forces are not specified as conservative or non-conservative.

There is no indication if the forces are constant or not.

The only requirement is that the forces must be internal to the system.

- This gives a hint about the power of this new model.

Newton's Second Law and Momentum

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Newton's Second Law:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

- This is the form in which Newton presented the Second Law.
- It is a more general form than the one we used previously.
- This form also allows for mass changes.

Linear momentum

Friday, 29 January, 2021 21:37

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A $3\ kg$ particle has a velocity of $(3\hat{\imath} - 4\hat{\jmath})\ m/s$.

- Find its x and y components of momentum.
- Find the magnitude and direction of its momentum.

$$\vec{P} = m\vec{v}$$

$$= 3(3\hat{\imath} - 4\hat{\jmath})$$

$$= (9\hat{\imath} - 12\hat{\jmath})kg \cdot m/s$$

***P9.4** We are given $m = 3.00$ kg and $\vec{\mathbf{v}} = (3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s.

(a) The vector momentum is then

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (3.00\text{ kg})\left[(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\text{ m/s}\right]$$
$$= (9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}$$

Thus, $\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$ and $\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$.

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00\text{ kg}\cdot\text{m/s})^2 + (12.0\text{ kg}\cdot\text{m/s})^2}$
$= \boxed{15.0\text{ kg}\cdot\text{m/s}}$

at an angle of

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

$$\theta = 360° - 53° = 307°$$

Momentum and kinetic energy

Saturday, 30 January, 2021 15:24

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
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An object has a kinetic energy of $275\,J$ and a momentum of magnitude $25\,kg.m/s$. Find the speed and mass of the object.

P9.2 $K = p^2/2m$, and hence, $p = \sqrt{2mK}$. Thus,

$$m = \frac{p^2}{2 \cdot K} = \frac{(25.0\ \text{kg} \cdot \text{m/s})^2}{2(275\ \text{J})} = \boxed{1.14\ \text{kg}}$$

and

$$v = \frac{p}{m} = \frac{\sqrt{2m(K)}}{m} = \sqrt{\frac{2(K)}{m}} = \sqrt{\frac{2(275\ \text{J})}{1.14\ \text{kg}}} = \boxed{22.0\ \text{m/s}}$$

Conservation of momentum

Saturday, 30 January, 2021 15:25

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

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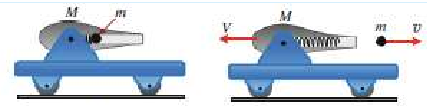
A cannon of mass $M = 1500\ kg$ shoots a projectile of mass $m = 100\ kg$ with a horizontal speed $v = 30\ m/s$, as shown. If the cannon can recoil freely on a horizontal ground, what is its recoil speed V just after shooting the projectile?



Before shooting After shooting

Solution: We take our system to be the cannon and the projectile, which both are at rest initially before shooting. When the trigger is pulled, the forces involved in the shooting are internal and hence cancel. During the very short time of shooting, we can assume that the external forces such as friction are very small compared to the forces exerted by the shooting. In addition, the external gravitational forces acting on the system have no components in the horizontal direction. Then the momentum conservation along the horizontal direction is:

$$P_i = P_f$$

The initial total horizontal momentum before the shooting is:

$$P_i = m \times 0 + M \times 0 = 0$$

The final total horizontal momentum after the shooting is:

$$P_f = mv + MV$$

Applying the conservation of total momentum $P_i = P_f$, we get:

$$V = -\frac{mv}{M} = -\frac{(100\,\text{kg})(30\,\text{m/s})}{1{,}500\,\text{kg}} = -2\,\text{m/s}$$

The minus sign indicates that the velocity and momentum of the cannon is opposite to that of the projectile. Since the cannon has a much larger mass than the projectile, its recoil speed is much less than that of the projectile.